Time decay of the remanent magnetization in the $\pm J$ spin glass model at $T=0$

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(Received 31 October 2000; published 18 May 2001)

Using the zero-temperature Metropolis dynamics, the time decay of the remanent magnetization in the $\pm J$ Edward-Anderson spin glass model with a uniform random distribution of ferromagnetic and antiferromagnetic interactions has been investigated. Starting from the saturation, the magnetization per spin m reveals a slow decrease with time, which can be approximated by a power law: $m(t)=m_\infty+(t/a_0)^{a_1}$, $a_1<0$. Moreover, its relaxation does not lead it into one of the ground states, and therefore the system is trapped in metastable isoenergetic microstates remaining magnetized. Such behavior is discussed in terms of a random walk that the system performs on its available configuration space.

DOI: 10.1103/PhysRevE.63.066111 PACS number(s): 05.50.+q, 75.10.Nr

I. INTRODUCTION

Spin glasses are systems that, at temperatures below the so-called glass-transition temperature T_g, find themselves in states with frozen disorder, i.e., no long-range patternlike order typical for ordered magnets is present. Their properties are determined by competing ferromagnetic and antiferromagnetic exchange interactions that are randomly distributed in the system. Both the competition among the different interactions between the magnetic moments and their random distribution all over a given system are likely to contribute significantly to such unusual glassy behavior. From the theoretical point of view, they can be discussed in terms of the coarse-grained free energy. Namely, due to accidental degeneracy present in such systems, below T_g their free-energy landscape becomes extremely rough, with many local minima corresponding to the same macroscopically observed properties but with entirely different microscopic states in the system phase space. The minima (valleys) are separated from each other by some energy barriers, and once a system finds itself in one of them, it might take a lot of time on laboratory time scales to transit to the others. Thus, the observed properties of spin glasses may only correspond to those of one single valley in which the system happens to be, and as a result, ergodicity is practically broken [1,2]. In other words, spin glasses can be seen as systems whose dynamics at low temperatures is extremely slow and whose properties measured in real experiments always correspond to situations out of equilibrium. Very good evidence for this nonstationary dynamics is the response of the system ac susceptibility to an oscillating field, i.e., its dependence both on time and frequency. Another example is the slow decay of remanent magnetizations with time [1,2]. The thermoremanent magnetization (TRM) is measured by cooling the sample in a nonzero magnetic field H from above the glass-transition temperature T_g to a temperature T below it and then switching off the field. The isothermal remanent magnetization (IRM) is measured by zero-field cooling of the sample in the same way as before, then turning on the field, and subsequently turning it off. In addition, in both cases the experimental results show that for small fields applied to the sample, the remanent magnetization is strongly affected by the so-called waiting time at which the sample is kept at constant temperature before the external field is changed [3]. According to the decay of both the remanent magnetization and the energy, the experimental results depend on the observation times after the field is switched off in all measurements. The above nonstationary dynamics has been described by a fair variety of functions. The most important ones include power law, logarithmic, stretched exponential, and others, and the question of judging which is the most universal is still far from decided [3–5]. Additionally, as some experiments indicate, the remanent magnetization decays so slowly with time that some nonzero remanence is still observed over macroscopic time scales, particularly at very low temperatures [6]. Relaxation time measurements in $CeNi_{0.8}Cu_{0.2}$ below the spin glass temperature 6 K show that the decay time increases drastically with a distinct tendency to a state with nonzero magnetization, which is higher the lower the temperature is [7]. The theoretical background of such a property is rather unclear [6].

One of the simplest theoretical models of spin glasses is the Edwards-Anderson (EA) model. Ising spins are located at each site of a lattice with randomly distributed ferromagnetic and antiferromagnetic interactions between the nearest neighbors. Such a model reveals most of the crucial features typical of real spin glasses including relaxation phenomena. Both the early papers [8,9] and the newer ones [10,11] on two- (2D) and three-dimensional (3D) models with a Gaussian distribution of bonds confirm that in a wide range of temperatures, a remanent magnetization occurs that decays very slowly with time t according to a power law $m(t) \sim t^{-\alpha}$ with $\alpha(T)\sim T$. It is also known that in case the couplings among spins may take only discrete values ($\pm J$ models) at sufficiently low temperatures, relaxation properties of such glasses are entirely different because of the existence of energy gaps in their energy spectra [11,12]. As a result, for models with bimodal distribution of interactions at temperatures well below T_g, a simple linear dependence of α on temperature T is not satisfied, and the functional form of the remanent magnetization decay is still rather far from being established. It has been mentioned that the function $\alpha(T)$ might go to zero faster than linear at low temperatures [11]. It has also been suggested that at finite small temperatures,

$m(t)$ should coincide with $m(t)$ at zero temperature for some region of time, and that the relaxation causes such a system to remain trapped in one metastable state with a finite remanent magnetization.

In this paper, we revisit the remanent magnetization decay of 2D $\pm J$ EA spin glass because we are interested in its extremely low-temperature relaxation properties where the power law with $\alpha(T) \sim T$ breaks down. We consider the limiting case of zero-temperature behavior in order to find out whether the remanence phenomena observed at such conditions could reflect the low-temperature properties of discrete systems at least qualitatively. We would also like to check out whether or not results obtained at $T=0$ could be treated as continuous extrapolations of those at low but finite temperatures, which, according to similar research done on the SK model [13], seems rather unlikely. Actually, in [13] it is shown that even the zero-temperature dynamics provides a decay of the magnetization that can be fitted by a power law with a constant exponent α. We carry out simulations using the zero-temperature dynamics, which has been successfully applied to various spin lattices including the persistence probability in the weakly disordered Ising model [14], hysteresis in the random-field Ising model on the Bethe lattice [15], the question of avalanches in spin systems [16], and others [17]. We investigate the system relaxation process towards low-energy states by plotting both energy and magnetization versus the zero-temperature Monte Carlo steps (MCS) per spin, which are treated as ''time units'' t. We discuss the influence of frustration on the nonequilibrium time properties by comparing them with corresponding properties for an unfrustrated system with pure antiferromagnetic interactions. Finally, we discuss the obtained results in terms of the system random walk on its configuration space.

II. MODEL AND SIMULATION

We use the Edward-Anderson spin glass model with a random and uniform distribution of discrete interactions $J_{ij} = \pm 1$ between the nearest neighbors all over a 2D square lattice with N sites and with periodic boundary conditions. The Hamilton function of such a system in an external magnetic field B is

$$H = -\sum_{i<j} J_{ij} S_i S_j - B \sum_i S_i , \tag{1}$$

where $S_i , S_j = \pm 1$ (up/down) are Ising spins, and the sum in the first term on the right-hand side runs over the nearest-neighboring lattice sites. The samples are prepared in such a way that the fraction of antiferromagnetic bonds is $p=0.5$. We study how the remanent magnetization decays with time in zero magnetic field starting from the system saturation state. The magnetization per spin is given by

$$m(t) = [N_+(t) - N_-(t)]/N, \tag{2}$$

where N_+ , N_- denote the number of spins up and down, respectively. The system relaxation process is simulated by applying a version of the zero-temperature Metropolis algorithm given by the following steps, cf. [15,17].

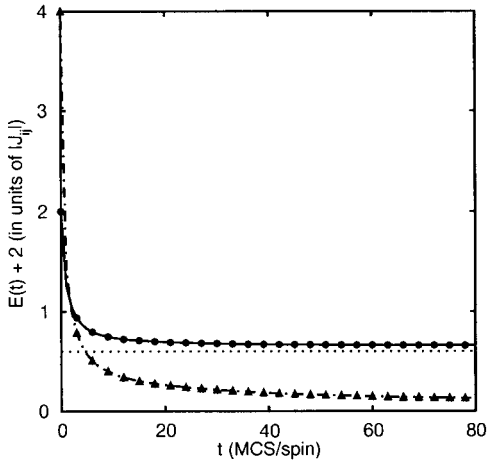

FIG. 1. Energy per spin E (in units of $|J_{ij}|$) versus time t (in units of Monte Carlo steps per spin) averaged over 30 independent runs in a frustrated $N=70\times70$ system (circles) and in an unfrustrated one of the same size with solely antiferromagnetic interactions (triangles). The dotted horizontal line represents the exact ground-state energy of the former. Energy axis is shifted by adding a term 2.

(i) Consider a sample in its saturation state (all spins up).

(ii) Pick a spin at random.

(iii) Flip it only if this process does not increase the energy.

(iv) Repeat steps (ii) and (iii) N times. So, one time unit is defined as one MC step per spin.

(v) Record magnetization and energy as functions of these time units.

(vi) Start again with step (ii) to find results for the following time step.

Thus, the above algorithm determines the system random walk on the configuration space in the direction of dropping energy. It should be mentioned that a different algorithm based on the Glauber dynamics is also used in the literature, cf. [14,18,19]. Then for $T=0$, step (iii) of the above-mentioned procedure is replaced by the following one: Flip it if this process decreases the energy and flip it with the probability $\frac{1}{2}$ if this process does not change the energy.

III. RESULTS

First we consider a special system of the size $N=70\times70$, for which the exact ground-state energy is calculated using a branch-and-cut algorithm by De Simone *et al.* [20]. Its energy and magnetization decays are compared with those of an unfrustrated one of the same size with solely antiferromagnetic interactions between the nearest neighbors. No qualitative differences in the energy relaxation can be seen, except for the fact that the ground-state energies of both systems are different (Fig. 1). On the other hand, with respect to the remanent magnetization decay, both systems are entirely different and the influence of randomness and frustration on the phenomena can be observed. While the unfrustrated system moves extremely fast to the low-energy

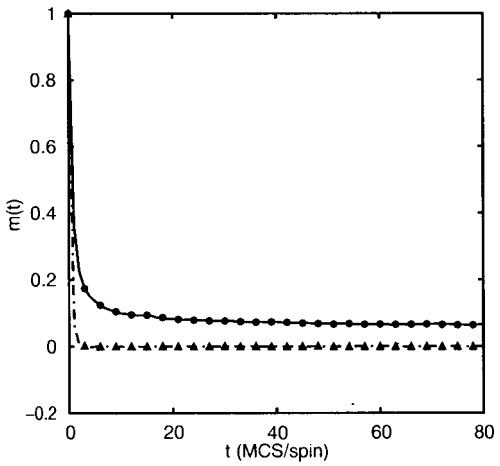

FIG. 2. Magnetization per spin m versus time t (in units of Monte Carlo steps per spin) averaged over 30 independent runs for the same systems as in Fig. 1.

unmagnetized states, the frustrated one relaxes more slowly and the isoenergetic states it finally explores still possess nonzero magnetization (Fig. 2). In that area, the remanent magnetization fluctuates around a constant value, which refers to simply flipping idle spins.

In order to describe the system dynamics more quantitatively, we have considered 10 samples of the size $N=50\times50$ with different distribution of bonds. Typically, we have performed around 2000 MC relaxation steps. In Fig. 3, we show the remanent magnetization $m(t)$ averaged over the samples and over 30 independent runs for each. The best fit for the dependence of the magnetization on time (for $t>0$) is obtained by

$$m(t)=m_\infty+\left(\frac{t}{a_0}\right)^{a_1}, \quad a_1<0. \tag{3}$$

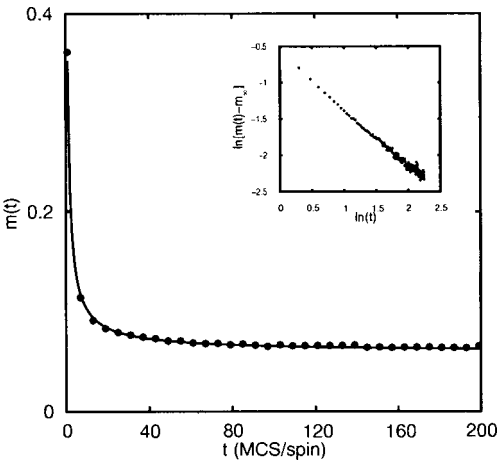

FIG. 3. Magnetization per spin m versus time t (in units of Monte Carlo steps per spin) averaged over 10 samples of the size $N=50\times50$ and over 30 runs for each. The continuous line represents the fit according to the power law (3). The inset shows $\ln[m(t)-m_\infty]$ versus $\ln(t)$.

The first term m_∞ corresponds to the average remanent magnetization of the system trapped in a subspace of low-lying isoenergetic states after a long time of decay, whereas the second term corresponds to the nonstationary dynamics. The parameters found are $m_\infty=0.059$, $a_0=0.23$, and $a_1=-0.83$. The inset of Fig. 3 shows $\ln[m(t)-m_\infty]$ versus $\ln(t)$ with m_∞ taken from the power-law fit (3). The observed dispersion of the computed points for large values of time t is due to their small fluctuations around m_∞. We have also tested some other functions with four free parameters to adjust the data. Among them, the stretched exponential law $m(t)=m_\infty+a_0\exp[-(t/a_1)^{a_2}]$ (for $t\geqslant0$), with $m_\infty=0.060$, $a_0=0.94$, $a_1=0.43$, and $a_2=0.35$, has proved reasonably close to it, however a bit worse than the power-law one.

IV. SUMMARY

We have investigated the remanent magnetization decay of a 2D $\pm J$ EA spin glass model at zero temperature. All the samples were initially at their saturation states and then allowed to relax towards states with lower energy using the zero-temperature Metropolis algorithm. The observed remanent magnetization decay in the frustrated system was much slower than in the unfrustrated one, whose remanent magnetization decreased rapidly to zero. After a number of MC steps, the walk of the random system on its configuration space practically became limited to subspaces of magnetized isoenergetic states. At that region, on the average, the remanent magnetization fluctuated without further decrease. Our calculations suggest that the time decay of the remanent magnetization can be very well represented by a simple power-law formula with three fitting parameters. From a microscopic point of view, the nonexponential relaxation phenomena in spin glasses can be discussed in terms of random diffusion on the available configuration space. It is suggested by simulations that stretched exponential relaxation behavior in glassy systems appears with the exponent a_2 going to $\frac{1}{3}$ when approaching a percolation transition in the configuration space, which is a multidimensional hypercube [21–23]. The same form of the magnetization decay in a 2D ferromagnetic Ising model has been reported and some dependence of the relaxation phenomena on the system dimension has been found as well [24–26]. In the so-called trap model, the system evolves among various traps with random ''trapping times.'' The traps are separated from each other by energy barriers that can be crossed by thermal excitations [27,28]. This kind of approach has been successfully used to study low-temperature aging of a system consisting of configurations with random energies [29]. However, since the dynamics of ours is athermal, there is no such barrier crossing in it and a purely entropic interpretation might be adapted instead [28,30]. Although for simulations at finite temperatures a mixture of ''energetic'' and ''entropic'' barriers is likely to contribute to the phenomena of slow dynamics, one of the effects vanishes with respect to the zero temperature simulations. The observed slowing down of the system relaxation could be qualitatively understood with the help of so-called entropic traps themselves. While the point in the configuration space randomly searches for available paths leading to

states with lower energy, their number decreases rapidly with time. As a result, we find that the lower the energy of the states is that the system has reached the longer is the time period needed to leave them. This phenomenon is also very consistent with the Markov theory [32,33]. It says that every finite Markov chain contains at least one absorbing set in which the system remains forever after falling in it (a good example of an absorbing set is the closure of one of the ground states). Moreover, the theory states that the probability of passing to one of them with ''time'' going to infinity tends to 1. That is, after a large number of MC steps, the available configuration space of the system becomes one of the absorbing sets, where the energy is kept constant and the magnetization fluctuates. A finite value of remanent magnetization m_∞ indicates that the absorbing set contains excited states rather than ground states. This is consistent with recent results on magnetic hysteresis at zero temperature [31]. All metastable states connected by one-spin flips without raising energy form a local ensemble of metastable states. When all these states belong to a relatively large value of m, the one-spin interconnection with the local ensemble of ground states can be prohibited.

Moreover, our simulations confirm that the observed tendency of the exponent $\alpha(T)$ [in the formula $m(t)\sim t^{-\alpha}$] to go to zero with dropping temperature seems to be in direct contradiction to the remanent magnetization decay at exactly $T=0$, for which α is finite. This also seems to indicate that the violations of the power law at very low temperatures might be due to the characteristic time τ divergence [11] rather than to the function $\alpha(T)$ going to zero faster than linear. Moreover, we think that another explanation could be suggested as well. Namely, it has been experimentally found that the remanent magnetization m_∞ decreases with increasing temperature [6,7]. Below a certain value of temperature T, the discrete structure of the system energy spectrum is likely to affect its properties significantly and m_∞ may become relevant. That means that at very low temperatures it should be contained in the relaxation law, and a plot $\ln[m(t)-m_\infty]$ versus $\ln(t)$ (instead of $\ln[m(t)]$ versus $\ln(t)$) could still remain linear.

ACKNOWLEDGMENTS

Support from Graduiertenkolleg ''Struktur- und Korrelationseffekte in Festkörpern'' is gratefully acknowledged. We thank D. Stauffer, J. Weißbarth, and M. Nogala for helpful discussions, and the authors of Ref. [20] for allowing us to use their computer data.

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